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Surface electro-elastic shear horizontal waves in a layered structure with a piezoelectric substrate and a hard dielectric layer

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Abstract

The existence and behaviour of surface electro-elastic shear horizontal waves in a layered structure consisting of a piezoelectric substrate of crystal class 6, 4, 6mm, or 4mm mechanically bonded at its upper surface to an elastic dielectric layer and bounded by an adjacent dielectric medium is considered when the shear bulk wave velocity in the elastic layer is greater than or equal to that in the substrate. The dispersion equation for the existence of the surface electro-elastic SH waves with respect to the phase velocity is obtained which includes all the above crystal classes i.e. the surface wave problems related to all these classes are presented in a single mathematical model. The investigation of the solutions of the dispersion equation is carried out and all the possible cases of the behaviour of the surface electro-elastic SH wave depending on the electro-mechanical coefficients of the layered structure are revealed.

Keywords: Surface electro-elastic SH wave; Layered piezoelectric structure; Electromechanical coupling coefficient

1. Introduction

The fact that elastic and electric fields in piezoelectric materials are interconnected gives rise to special effects, particularly a surface wave which is called the Bleustein–Gulyaev wave and exists only in piezoelectric materials (Bleustein, 1968; Gulyaev, 1969). The penetration depth of this wave is 10–100 wavelengths which restricts its application mainly to microwave technology. However the penetration depth can be reduced if a thin layer of an elastic material is bonded to the surface of the piezoelectric material. In such layered structures an surface electro-elastic Love wave can propagate the characteristics of which will depend on the piezoelectric properties of the crystal. Conditions for the existence of surface electro-elastic Love waves in a piezoelectric material of class 6mm carrying a metal layer of finite thickness are developed in Curtis and Redwood (1973). Many papers are devoted to the investigation of the propagation of surface electro-elastic Love waves in certain piezoelectric materials (of classes 6mm and 4mm) carrying a dielectric elastic layer, with or without

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initial stresses (Jin et al., 2000; Liu et al., 2001; Wang et al., 2001; Jin et al., 2005; Qian et al., 2004). The properties of a structure where the surface layer is a piezoelectric material identical to that of the substrate but with opposite polarization with or without initial stresses is considered in Jin et al. (2002), Jin et al. (2001), Liu et al. (2003).

Danoyan and Piliposian (2007) have discussed the existence and behaviour of surface electro-elastic Love waves in a more general range of piezoelectric materials with a soft dielectric surface layer where the shear bulk wave velocity in the surface layer is less than that in the substrate. The results obtained cover all piezoelectric crystals of symmetry classes 6, 4, 6mm, 4mm, 622 and 422.

It has been shown (Hanhua and Xingjiao, 1993) that surface electro-elastic SH waves can also propagate in certain dielectric/piezoelectric layered structures with a hard surface layer, when the shear bulk wave velocity in the surface layer is greater than or equal to that in the piezoelectric substrate. In this paper we consider the existence and behaviour of such waves in a wider class of piezoelectric materials. We show that such waves can only exist when the electromechanical coupling coefficient is positive, excluding piezoelectric materials of crystal classes 622 and 422 which have a negative coupling coefficient (Danoyan and Piliposian, 2007). For this reason our investigations in this paper will be concerned with the existence and behaviour of surface electro-elastic SH waves in layered structures with a hard dielectric surface layer and a piezoelectric substrate of crystal class 6, 4, 6mm or 4mm.

2. The statement of the problem

A layered piezoelectric structure consisting of an elastic isotropic dielectric layer of thickness h rigidly linked to the piezoelectric half-space substrate either of classes 6, 4, 6mm or 4mm is considered (Fig. 1). The dielectric layer can be any isotropic material, or any anisotropic material with the same anisotropy as the substrate. The Ox_3 axis of the coordinate system $Ox_1x_2x_3$ is directed along the main direction of the piezoelectric substrate and the plane $x_1 = 0$ occupies the boundary between the layer and the substrate. The Ox_1 axis points down into the substrate.

The domain $x_1 < -h$ is assumed to be either a vacuum or it is occupied by a dielectric medium without an acoustic contact with the layer. The layer surface $x_1 = 0$ is electrically open, the surface $x_1 = -h$ is electrically open or shorted and is free of external forces (mechanically free).

We consider an antiplane problem (Danoyan and Piliposian, 2007):

$$\begin{cases} U_1 \equiv 0, \quad U_2 \equiv 0, \quad U_3 = u(x_1, x_2, t), \ -h \leqslant x_1 < +\infty, \\ \varphi = \varphi(x_1, x_2, t), \ -\infty < x_1 < +\infty. \end{cases} \tag{2.1}$$

As has been shown in Danoyan and Piliposian (2007), in the present setting the coupled electromechanical field equations for the piezoelectric substrate, dielectric layer and dielectric medium can be presented in the following form:

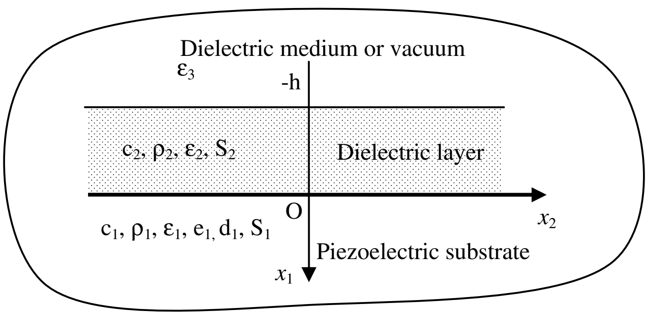

Fig. 1. The layered half-space for dielectric layer and piezoelectric substrate.

$$1. \quad \nabla^2 u_1 = \frac{1}{S_1^2}\frac{\partial^2 u_1}{\partial t^2}, \quad \nabla^2 \varphi_1' = 0, \qquad \text{in the substrate } x_1 > 0 \tag{2.2}$$

$$2. \quad \nabla^2 u_2 = \frac{1}{S_2^2}\frac{\partial^2 u_2}{\partial t^2}, \quad \nabla^2 \varphi_2 = 0, \qquad \text{in the layer } -h < x_1 < 0 \tag{2.3}$$

$$3. \quad \nabla^2 \varphi_3 = 0, \qquad \text{in the domain } \quad x_1 < -h. \tag{2.4}$$

The boundary conditions are

$$\begin{cases} u_1 = u_2, \bar{e}_1 u_1 + \varphi_1' = \varphi_2, -\varepsilon_1 \frac{\partial \varphi_1'}{\partial x_1} + d_1 \frac{\partial u_1}{\partial x_2} = -\varepsilon_2 \frac{\partial \varphi_2}{\partial x_1}, \\ \bar{c}_1 \frac{\partial u_1}{\partial x_1} + e_1 \frac{\partial \varphi_1'}{\partial x_1} - d_1 \bar{e}_1 \frac{\partial u_1}{\partial x_2} - d_1 \frac{\partial \varphi_1'}{\partial x_2} = c_2 \frac{\partial u_2}{\partial x_1}, \end{cases} \quad \text{when } x_1 = 0 \tag{2.5}$$

$$\varphi_2 = \varphi_3, \quad \frac{\partial u_2}{\partial x_1} = 0, \quad -\varepsilon_2 \frac{\partial \varphi_2}{\partial x_1} = -\varepsilon_3 \frac{\partial \varphi_3}{\partial x_1}, \qquad \text{when } x_1 = -h. \tag{2.6}$$

The attenuation conditions for surface waves at $x \to \pm\infty$ are

$$\begin{cases} u_1 \to 0, \varphi_1' \to 0, \text{for} \quad x_1 \to +\infty, \\ \varphi_3 \to 0, \quad \text{for} \quad x_1 \to -\infty, \end{cases} \tag{2.7}$$

where

$$\varphi_1' = \varphi_1 - \bar{e}_1 u_1, \quad S_1 = \sqrt{\bar{c}_1/\rho_1}, \quad S_2 = \sqrt{c_2/\rho_2}, \quad \bar{c}_1 = c_1(1+\chi_1^2), \quad \chi_1^2 = e_1^2/\varepsilon_1 c_1, \quad c_1 = c_{44}^{(1)},$$
$$c_2 = c_{44}^{(2)}, \quad d_1 = e_{14}^{(1)}, \quad e_1 = e_{15}^{(1)}, \quad \bar{e}_1 = e_1/\varepsilon_1, \quad \varepsilon_1 = \varepsilon_{11}^{(1)},$$

and u_i are the mechanical displacements, φ_i the electric potentials, ρ_i the mass density of the medium, $c_{kj}^{(i)}, e_{kj}^{(i)}, \varepsilon_{kj}^{(i)}$ are the elastic, piezoelectric and dielectric constants (superscripts and subscripts (i), $i = 1, 2, 3$ indicate that the value belongs to the substrate, the layer and the dielectric medium respectively), S_1 and S_2 are the velocities of shear bulk waves in the substrate and the layer, χ_1 the electromechanical coupling coefficient for the shear bulk wave.

3. Solution of the problem. The dispersion equation of the surface wave

The solution of the boundary problem (2.2)–(2.6) as a plane harmonic wave satisfying the attenuation conditions (2.7) has the following form (Danoyan and Piliposian, 2007):

$$\begin{cases} u_1 = U_{01} e^{-p\beta_1(V)x_1} e^{i(px_2 - \omega t)} \\ \varphi_1 = [U_{01}\bar{e}_1 e^{-p\beta_1(V)x_1} + \Phi_{01} e^{-px_1}] e^{i(px_2-\omega t)}, \end{cases} \quad x_1 > 0, \tag{3.1}$$

$$\begin{cases} u_2 = [U_{02}^+ e^{-p\gamma_2(V)x_1} + U_{02}^- e^{p\gamma_2(V)x_1}] e^{i(px_2-\omega t)} \\ \varphi_2 = [\Phi_{02}^+ e^{px_1} + \Phi_{02}^- e^{-px_1}] e^{i(px_2-\omega t)} \end{cases} \quad -h < x_1 < 0, \tag{3.2}$$

$$\varphi_3 = \Phi_{03} e^{px_1} e^{i(px_2-\omega t)}, \qquad -\infty < x_1 < -h. \tag{3.3}$$

Here $U_{01}, \Phi_{01}, U_{02}^+, U_{02}^-, \Phi_{02}^+, \Phi_{02}^-, \Phi_{03}$ are arbitrary constants, and $\beta_1(V)$ and $\gamma_2(V)$ are attenuation coefficients, where

$$\beta_1(V) = \sqrt{1 - (V^2/S_1^2)}, \quad \gamma_2(V) = \sqrt{1 - (V^2/S_2^2)}. \tag{3.4}$$

Further, it is assumed that $\omega > 0$ and $p > 0$ and the phase velocity is

$$V = \omega/p. \tag{3.5}$$

From the attenuation condition (2.7) for $x_1 \to +\infty$, it follows that $\beta_1(V)$ is positive and therefore a necessary condition for a surface wave to exist is

$$0 < V_S < S_1, \tag{3.6}$$

where V_S is the velocity of the surface electro-elastic SH wave.

We consider here the case of a hard layer i.e. when

$$S_1 \leqslant S_2. \tag{3.7}$$

It follows from (3.6) and (3.7) that $\gamma_2(V)$ is positive.

Substituting solutions (3.1)–(3.3) into the boundary conditions (2.5), (2.6) yields a homogeneous system of algebraical equations for the unknown amplidudes. The existence condition for the solution of this system of equations gives the dispersion equation of the surface wave (Danoyan and Piliposian, 2007),

$$\beta_1(V) = -c\gamma_2(V)\tanh[k\gamma_2(V)] + R(k), \tag{3.8}$$

where the following notations are introduced:

$$R(k) = \frac{R_1^2\bar{\varepsilon}_2(\bar{\varepsilon}_2\tanh k + 1) - K_1^2\bar{\varepsilon}_1(\bar{\varepsilon}_2 + \tanh k)}{\bar{\varepsilon}_2(\bar{\varepsilon}_2\tanh k + 1) + \bar{\varepsilon}_1(\bar{\varepsilon}_2 + \tanh k)}, \quad \text{(electrically open case)} \tag{3.9}$$

$$R(k) = \frac{R_1^2\varepsilon_2 - K_1^2\varepsilon_1\tanh k}{\varepsilon_2 + \varepsilon_1\tanh k}, \quad \text{(electrically shorted case)} \tag{3.10}$$

$$c = c_2/\bar{c}_1, \quad \bar{\varepsilon}_1 = \varepsilon_1/\varepsilon_3, \quad \bar{\varepsilon}_2 = \varepsilon_2/\varepsilon_3,$$
$$R_1^2 = e_1^2/\varepsilon_1\bar{c}_1, \quad K_1^2 = d_1^2/\varepsilon_1\bar{c}_1. \tag{3.11}$$

$$R_0 \equiv R(0) = \frac{R_1^2 - K_1^2\bar{\varepsilon}_1}{1+\bar{\varepsilon}_1} = \frac{1}{\varepsilon_1\bar{c}_1}\frac{e_1^2 - d_1^2\bar{\varepsilon}_1}{1+\bar{\varepsilon}_1}, \tag{3.12}$$

$$R_\infty \equiv R(\infty) = \frac{R_1^2\bar{\varepsilon}_2 - K_1^2\bar{\varepsilon}_1}{\bar{\varepsilon}_2+\bar{\varepsilon}_1} = \frac{1}{\varepsilon_1\bar{c}_1}\frac{e_1^2\bar{\varepsilon}_2 - d_1^2\bar{\varepsilon}_1}{\bar{\varepsilon}_2+\bar{\varepsilon}_1}. \tag{3.13}$$

Here R_1^2 and K_1^2 are the electromechanical coupling coefficients of shear bulk waves, and $R(k)$ is the electromechanical coupling coefficient of the surface waves. Note that the electromechanical coupling coefficient (3.10) can be obtained from (3.9) by assuming $\varepsilon_3 >> \varepsilon_1$ and $\varepsilon_3 >> \varepsilon_2$.

Note also that for the known piezoelectric materials

$$|R(k)| < 1. \tag{3.14}$$

4. The electro-mechanical coupling coefficient

It follows from the dispersion Eq. (3.8) that only one mode of the surface electro-elastic SH wave propagates in a layered structure with a piezoelectric substrate and a hard dielectric layer. It exists only for those piezoelectric substrates and the values of the parameter k for which $R(k) > 0$. Detailed investigation of the electromechanical coupling coefficient depending on the electro-mechanical parameters of the layered structure is carried out in Danoyan and Piliposian (2007). It is shown that for electrically open case the conditions for electro-mechanical parameters when $R(k) > 0$ are as follows:

$$1.\ \bar{\varepsilon}_2 > 1,\ d_1 = 0 \quad \text{or} \quad \bar{\varepsilon}_2 > 1, \quad \bar{\varepsilon}_1 < \varepsilon_*, \quad \text{where} \quad \varepsilon_* = (e_1/d_1)^2, \tag{4.1}$$

$R(k) > 0, k \in [0,\infty], R(k)$ is increasing monotonically from $R_0 > 0$ to $R_\infty > 0$(see (3.12) and (3.13)).

$$2.\ \bar{\varepsilon}_2 < 1,\ d_1 = 0 \quad \text{or} \quad \bar{\varepsilon}_2 < 1, \quad \bar{\varepsilon}_1 < \bar{\varepsilon}_2\varepsilon_*, \tag{4.2}$$

$R(k) > 0, k \in [0,\infty], R(k)$ is decreasing monotonically from $R_0 > 0$ to $R_\infty > 0$.

$$3.\ \bar{\varepsilon}_2 = 1,\ d_1 = 0 \quad \text{or} \quad \bar{\varepsilon}_2 = 1, \quad \bar{\varepsilon}_1 < \varepsilon_*, \tag{4.3}$$

$R(k) \equiv R_0 = \mathrm{R}_\infty = \text{const} > 0, k \in [0,\infty]$.

It follows from (4.1)–(4.3) that $R(k) > 0$ for some piezoelectric crystals of classes 6, 4 and all crystal classes 6 mm and 4 mm.

For the electrically shorted case the function $R(k)$ is always monotonically decreasing ($R_0 > R_\infty > 0$) and $R(k) > 0$ if (Danoyan and Piliposian, 2007):

1. $e_1 > 0$ and $d_1 = 0$ (crystal classes 6 mm and 4 mm), (4.4)
2. $e_1 > 0$, $d_1 > 0$ and $\varepsilon_1 < \varepsilon_2\varepsilon_*$ (crystal classes 6 and 4). (4.5)

5. The case of a hard layer ($S_2 > S_1$)

The existence and behaviour of the surface electro-elastic SH wave depends on the electro-mechanical parameters of the structure as well as on the existence of the Bleustein–Gulyaev wave which propagates in a piezoelectric half-space in the absence of a layer when $R_0 > 0$. It is defined from the dispersion Eq. (3.8) when $k = 0$ and has the following form:

$$V_{BG} \equiv S_1\sqrt{1 - R_0^2} < S_1. \tag{5.1}$$

According to the necessary condition of the existence of a surface wave (3.6) the velocity of the surface wave will satisfy the following condition:

$$0 < V_S < S_1 \leqslant S_2. \tag{5.2}$$

5.1

When $V = S_1$ (the surface electro-elastic SH wave does not exist) we get from the dispersion Eq. (3.8)

$$\tanh[k\gamma_2(S_1)] = \mu_k, \tag{5.3}$$

where

$$\mu(k) = R(k)/c\gamma_2(S_1). \tag{5.4}$$

Consider first the case (4.3) when $R(k) = R_0 = R_\infty = \text{const} > 0$. The following cases are possible here

1. $\mu_{cons} < 1$, (5.5)
2. $\mu_{cons} = 1$, (5.6)
3. $\mu_{cons} > 1$, (5.7)

where

$$\mu_{cons} = R_0/c\gamma_2(S_1) = \text{const}. \tag{5.8}$$

In the case (5.5) the surface electro-elastic SH wave velocity $V(k)$ monotonically increases from $V(0) = V_{GB}$ to $V_{qui} = S_1$, when k changes from $k = 0$ to $k = k_{qui}$, where the critical value $k = k_{qui}$ is defined from the following equation:

$$\tanh(k_{qui}\gamma_2(S_1)) = \mu_{qui} = \mu_{cons}. \tag{5.9}$$

When $k = k_{qui}$ the solution of the dispersion Eq. (3.8) constitutes a shear bulk wave. When $k > k_{qui}$ Eq. (3.8) does not have solutions and the interval $k > k_{qui}$ becomes a quiescent zone. The dispersion curve in this case is shown in Fig. 2a.

It is easy to see, that in the case (5.6) $k_{qui} = \infty$ and the quiescent zone disappears. The velocity of the surface wave increases from $V = V_{BG}$ to $V = S_1$ when k increaces from 0 to ∞. When $k \to \infty$ the limiting wave exists which is a shear bulk wave. The behaviour of the dispersion curve is shown in Fig. 2b.

In the case (5.7) it is clear that (5.3) can not take place which means that the surface electro-elastic SH wave velocity can not rich the value S_1. It increasingly approaches to some value V_∞ for $k \to \infty$ which can be defined from the following equation:

$$\beta_1(V_\infty) = -c\gamma_2(V_\infty) + R_\infty. \tag{5.10}$$

It can be shown that Eq. (5.10) has a solution in the interval $[0, S_1]$. If we write the dispersion Eq. (3.8) in the following form

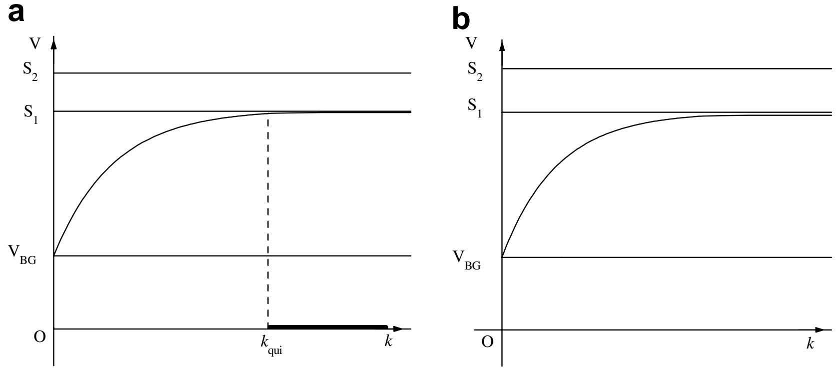

Fig. 2. Two possible patterns of behaviour of the dispersion curves for a hard layer ($S_2 > S_1$).

$$g(k, V) \equiv \beta_1(V) - [-c\gamma_2(V)\tanh[k\gamma_2(V)] + R(k)], \tag{5.11}$$

and define the values of $g(k, V)$ at the edges of the interval $V \in [0, S_1]$ when $k \to \infty$, we will have

$$g(\infty, 0) = 1 + c - R > 0, \tag{5.12}$$

and taking into account (5.7) we can write

$$g(\infty, S_1) = c\gamma_2(S_1) - R < 0. \tag{5.13}$$

It follows from (5.12) and (5.13) that $g(\infty, V) = 0$ at some point $V_\infty \in [0, S_1]$ and therefore the limiting value V_∞ is the solution of Eq. (5.10). The behaviour of the dispersion curve is shown in Fig. 3a.

5.2

Consider the case when the parameters of the layered structure satisfy the conditions (4.1), where the electro-mechanical coupling coefficient is positive and increasing monotonically from $R_0 = 0$ to $R_\infty > 0$. Here the following cases are possible:

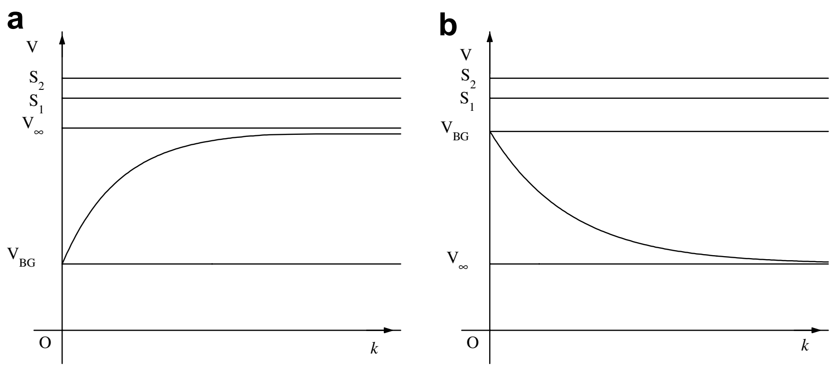

Fig. 3. Two possible patterns of behaviour of the dispersion curves for a hard layer ($S_2 > S_1$).

1. $\mu_\infty < 1,$ (5.14)
2. $\mu_\infty = 1,$ (5.15)
3. $\mu_\infty > 1,$ (5.16)
   $a.\quad \mu_0 \geqslant 1,\quad P(V_{BG}) > 0,$ (5.17)
   $b.\quad \mu_0 \geqslant 1,\quad P(V_{BG} < 0,$ (5.18)
   $c.\quad \mu_0 \geqslant 1,\quad P(V_{BG}) = 0, V_{BG}^2 < S_2^2(1 - R_0^/ /c),$ (5.19)
   $d.\quad \mu_0 \geqslant 1,\quad P(V_{BG}) = 0, V_{BG}^2 > S_2^2(1 - R_0^/ /c),$ (5.20)
4. $\mu_\infty > 1,\quad \mu_0 < 1,$ (5.21)

where

$$\mu_0 = \mu(0),\quad \mu_\infty = \mu(\infty), \tag{5.22}$$

and $P(V_{BG})$ is defined in (5.24).

The dispersion curves for cases (5.14) and (5.15) are shown in Fig. 2a, b.

In the case (5.16) it is obvious that $V_\infty < S_1$ but the question arises whether (a) $V_\infty > V_{BG}$, (b) $V_\infty < V_{BG}$ or c) $V_\infty = V_{BG}$. If $V_\infty > V_{BG}$ then $\beta_1(V_\infty) < \beta_1(V_{BG})$ and it follows from (3.8) that $R_0 - R_\infty + c\gamma_2(V_\infty) > 0$. As in this case $\gamma_2(V_{BG}) > \gamma_2(V_\infty)$ then

$$R_0 - R_\infty + c\gamma_2(V_{BG}) > R_0 - R_\infty + c\gamma_2(V_\infty) > 0. \tag{5.23}$$

Substituting $V_{BG}^2 = S_1^2(1 - R_0^2)$ in (5.23) we obtain the following condition

$$P(V_{GB}) = R_0 - R_\infty + cS_1S_2^{-1}\sqrt{R_0^2 - \beta_1^2(S_2)} > 0. \tag{5.24}$$

Thus if $\mu_\infty > 1$ with $\mu_0 \geqslant 1$ and $P(V_{BG}) > 0$ ((5.16) and (5.17)) then $V_\infty > V_{BG}$ and the dispersion curve behaves as shown in Fig. 3a. Following the same discussion it can be shown that if $\mu_\infty > 1$ with $\mu_0 \geqslant 1$ and $P(V_{BG}) < 0$ ((5.16) and (5.18)) then $V_\infty < V_{BG}$. The dispersion curve is shown in Fig. 3b.

In the case when $\mu_\infty > 1$, $\mu_0 \geqslant 1$ and $V_\infty = V_{BG}$ which can take place when $P(V_{BG}) = 0$ the surface wave has an extremum point which can be defined from the equation $F\prime(k) = 0$, where

$$F(k) = \beta_1(V) + c\gamma_2(V)\tanh[k\gamma_2(V)] - R(k). \tag{5.25}$$

If $F'(0) > 0$ the dispersion curve will have a maximum point subject to the following condition ((5.16) and (5.19))

$$V_{BG}^2 < S_2^2(1 - R_0^/ /c), R_0^/ \equiv R^/(0) > 0. \tag{5.26}$$

The dispersion curve is shown in Fig. 4a.

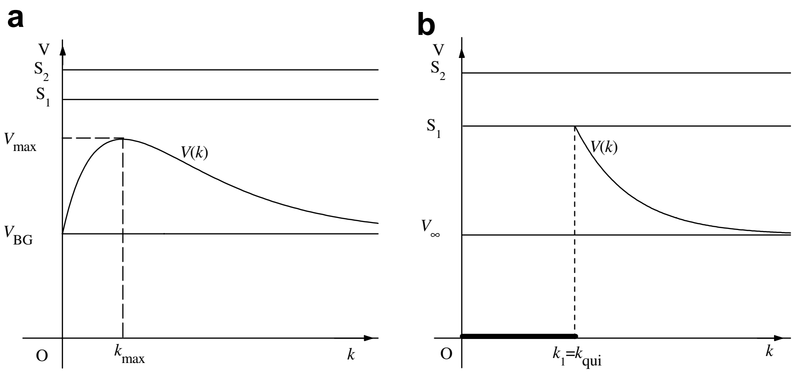

Fig. 4. Two possible patterns of behaviour of the dispersion curves for a hard layer ($S_2 > S_1$).

If $F'(0) < 0$ the dispersion curve will have a minimum point if $V_{BG}^2 > S_2^2(1 - R_0^{/}/c)$ ((5.16) and (5.20)). The dispersion curve is shown in Fig. 5a.

In the case (5.21) the dispersion curve is shown in Fig. 2a.

Following the same discussion it can be shown that two patterns of the dispersion curve shown in Fig. 5 are possible if $R(k) \geqslant 0$.

5.3

When the parameters of the layered structure satisfy the conditions (4.2), the electromechanical coupling coefficient is positive and decreasing monotonically from $R_0 = 0$ to $R_\infty > 0$. Here again depending on the three possible cases (5.14)–(5.16), the correspondent dispersion curves are shown in Figs. 2a,b, 3a.

5.4

It is easy to see that if $R(k)$ changes sign at some point $k = k^*$ the surface wave exists only for those values of k for which $R(k) > 0$. If $R(k)$ changes sign from a negative value to a positive value ($R_0 < 0, R(k_*) = 0, R_\infty > 0$), (which takes place for layered structures with $\bar{\varepsilon}_2 > 1, \varepsilon_* < \bar{\varepsilon}_1 < \bar{\varepsilon}_2\varepsilon_*$, (Danoyan and Piliposian, 2007), then in the case of (5.16) there is a quiescent zone for $0 \leqslant k < k_* < k_{qui}$, where a shear bulk wave corresponds to k_{qui}, which propagates with the velocity $V(k_{qui}) = S_1$. With k increasing it becomes a surface wave, with the velocity decreasing from $V(k_{qui}) = S_1$ to $V_\infty = V(\infty) < S_1$. The limiting value for $k \to \infty$ exists and has the velocity $V = V_\infty < S_1$. The dispersion curve is shown in Fig. 4b.

In the case (5.15) the dispersion Eq. (3.8) has a solution only for $k = \infty$. It describes a shear-surface wave propagating with a velocity $V = S_1$ in a structure consisting of two half spaces. In the case (5.14) the dispersion Eq. (3.8) does not have a solution.

If $R(k)$ changes sign from positive value to a negative value ($R_0 > 0, R(k_*) = 0, R_\infty < 0$), (which takes place for $\bar{\varepsilon}_2 < 1$ and $\bar{\varepsilon}_2\varepsilon_* < \bar{\varepsilon}_1 < \varepsilon_*$ (Danoyan and Piliposian, 2007)) the surface electro-elastic SH wave exists for $0 \leqslant k \leqslant k_{qui} < k_*$ with the velocity increasing from $V(0) = V_{GB}$ to $V(k_{qui}) = S_1$. A quiescent zone occurs for $k > k_{qui}$. The dispersion curve in this case is shown in Fig. 2a.

6. The case when $S_2 = S_1$

If $S_2 = S_1$ the dispersion Eq. (3.8) takes the following form:

$$\beta_1(V)[1 + c\tanh(k\beta_1(V))] = R(k), \quad 0 < V < S_1 = S_2. \tag{6.1}$$

For the layered structures with $R(k) > 0$ ($0 \leqslant k < \infty$) the surface electro-elastic SH wave exists for all values of k and propagates with the velocity $V_{BG} = S_1\sqrt{1 - R_0^2}$ when $k = 0$ and $V_\infty = S_1\sqrt{1 - R_\infty^2/(1+c)^2} < S_1$

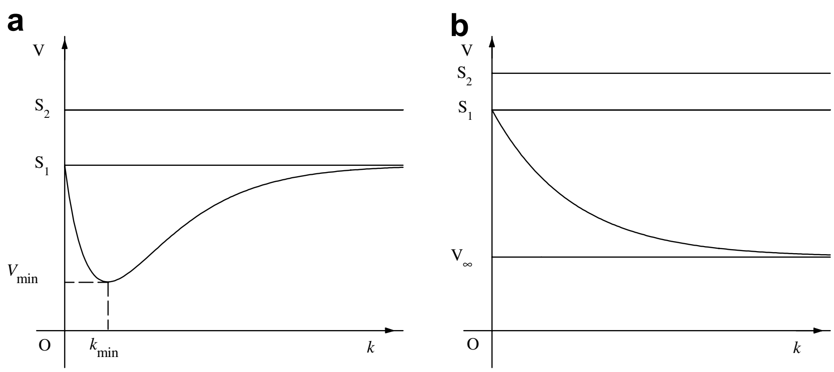

Fig. 5. Two possible patterns of behaviour of the dispersion curves for a hard layer ($S_2 > S_1$).

when $k \to \infty$. The behaviour of the surface electro-elastic SH wave will depend on the relationship between V_{BG} and V_∞.

6.1

If $R(k) > 0$ and monotonically increasing from R_0 to R_∞ (see (4.1)) the following cases are possible:

1. $R_\infty < (1+c)R_0, (V_{BG} < V_\infty),$ (6.2)
2. $R_\infty > (1+c)R_0, (V_{BG} > V_\infty),$ (6.3)
3. $R_\infty = (1+c)R_0, (V_{BG} = V_\infty), \quad V_{BG} < S_2\sqrt{1 - R_0'/c},$ (6.4)
4. $R_\infty = (1+c)R_0, (V_{BG} = V_\infty), \quad V_{BG} > S_2\sqrt{1 - R_0'/c}.$ (6.5)

The corresponding dispersion curves are shown in Figs. 6, 7.

6.2

If $R(k) > 0$ and monotonically decreasing from R_0 to R_∞ or $R(k) = const > 0$ (see (4.2) and (4.3)) the surface electro-elastic SH wave propagates with the velocity increasing from V_{BG} to V_∞. The dispersion curve in these cases are shown in Fig. 6a.

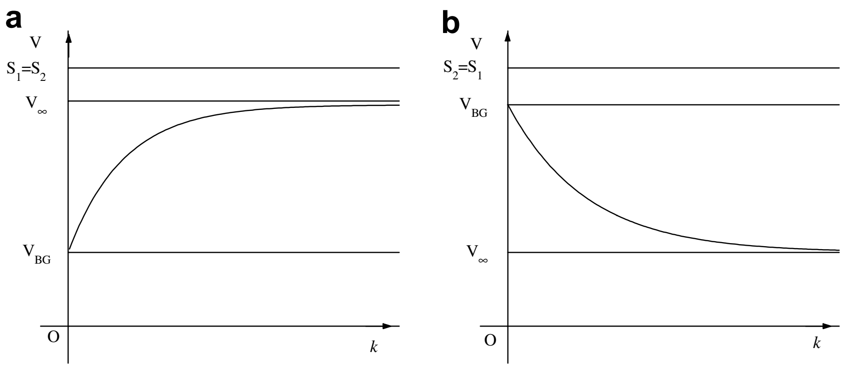

Fig. 6. Two possible patterns of behaviour of the dispersion curves for a hard layer ($S_2 = S_1$).

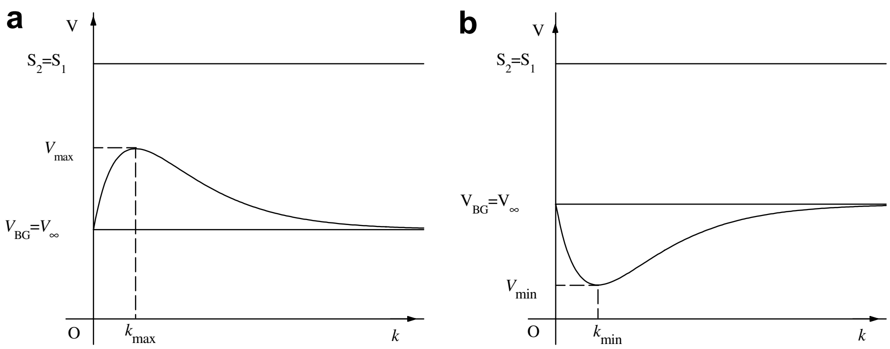

Fig. 7. Two possible patterns of behaviour of the dispersion curves when $S_2 = S_1$.

6.3

If $R(k)$ changes sign from a negative value to a positive value ($R_0 < 0, R(k_*) = 0, R_\infty > 0$) a quiescent zone occurs in the interval $0 \leqslant k < k_{qui} = k_*$, where the shear wave corresponds to the value of k_{qui} with $V(k_{qui}) = S_1$. With k increasing it becomes a surface wave and the velocity decreases from $V(k_{qui}) = S_1$ to $V_\infty < S_1$. The limiting surface wave for $k \to \infty$ in this case exists and the dispersion curve is shown in Fig. 8a.

If $R(k)$ changes sign from positive value to a negative value ($R_0 > 0, R(k_*) = 0, R_\infty < 0$) the surface electro-elastic SH wave exists in the interval $0 \leqslant k \leqslant k_{qui} = k_*$ with the velocity increasing from $V(0) = V_{GB}$ to $V(k_{qui}) = S_1$. A quiescent zone occurs for $k > k_*$. The dispersion curve is shown in Fig. 8b.

Following the same discussion it can be shown that if $R(k) \geqslant 0$ the surface electro-elastic SH wave behaves as shown in Fig. 9.

The possible cases of the behaviour of the surface electro-elastic SH wave for electrically shorted boundary conditions are shown in Figs. 2a,b, 3a.

7. Conclusion

Surface electro-elastic SH wave can propagate in layered structures with a piezoelectric substrate of crystal classes 6, 4, 6mm and 4mm and a hard dielectric layer. The component waves in the layer are inhomogeneous and the dependence of the phase velocity on the relative thickness of the layer is single valued. That means that the surface electro-elastic SH waves have only one mode.

For piezoelectric substrates of crystal classes 6mm and 4mm and some of crystal classes 6 and 4 specified in the paper this type of the surface electro-elastic SH wave has a so-called quiescent zone i.e. the surface electro-elastic SH wave does not exist for certain values of the relative thickness of the layer (Figs. 2a, 4b, 8a,b)

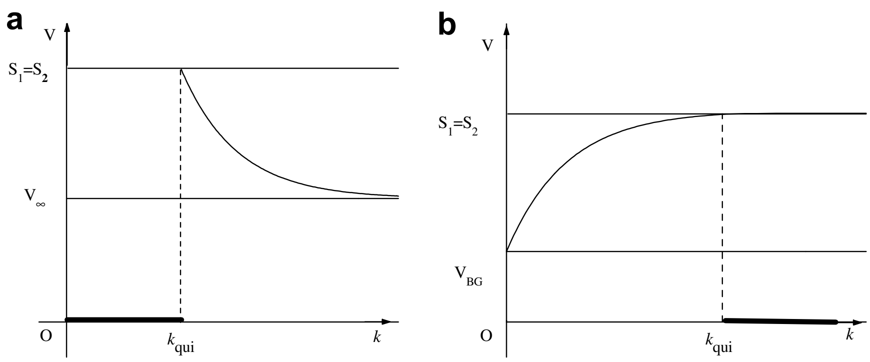

Fig. 8. Two possible patterns of behaviour of the dispersion curves when $S_2 = S_1$.

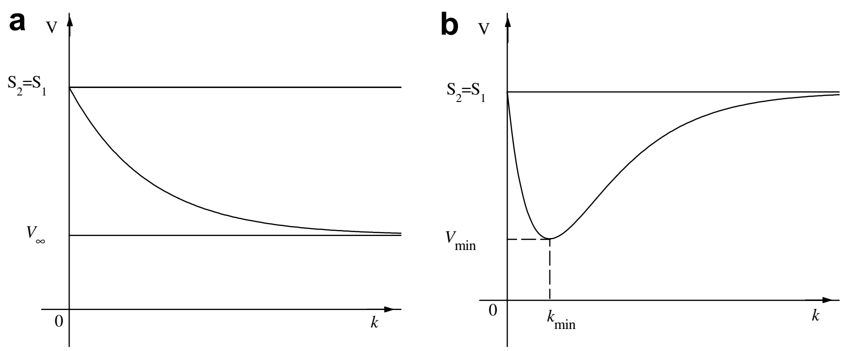

Fig. 9. Two possible patterns of behaviour of the dispersion curves when $S_2 = S_1$.

For certain conditions on the piezoelectric and dielectric parameters of the piezoelectric substrate the surface electro-elastic SH wave velocity decreases monotonically throughout the whole interval of existence, like in the elastic Love wave case.

Depending on the relationship between the shear bulk wave velocity of the layer and the Bleustein–Gulyaev wave velocity, cases are found when it does not change monotonically, as in the elastic Love wave case. It reaches an extremum value (maximum or minimum) for some intermediate value of the relative thickness (Figs. 4a, 5a, 7a,b, 9b). In some cases the surface electro-elastic SH wave velocity increases monotonically throughout the whole interval of existence (Figs. 2a,b, 3a, 6a, 8b).

In the absence of a layer the surface electro-elastic SH wave of the gap type becomes the Bleustein–Gulyaev wave if $R_0 > 0$. It becomes either a surface or a shear-surface wave or disappears for two half spaces.

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